Exercise 1. Are the following functions linear? (Remember, \( f \) is linear if i) for all pairs of vectors \( u, v \), we have \( f(u + v) = f(u) + f(v) \), and ii) for every vector \( v \) and scalar \( c \), \( f(cv) = cf(v) \).

\( a) \) \( f(x) = 3x \), \( b) \) \( g(x) = 2x - 3 \), \( c) \) \( h(x) = x^2 \), \( d) \) \( k(x, y) = x + y \),

\( e) \) \( l(x, y) = (x + y, x - y) \), \( f) \) \( m(x, y) = (x, 1) \), \( g) \) \( n(x, y) = (0, y) \).

Exercise 2. Is it true that a linear transformation \( f \) always fixes the origin, i.e., \( f(0) = 0 \)?

Exercise 3. The goal of this exercise is getting an intuition about the geometric meaning of linear transformations and how matrices describe them. Go to the website “Linear Transformation in 2 dimensions” (http://mathinsight.org/applet/linear_transformation_2d) where you will find the applet we were playing around with in the end of the class. Try out the following input matrices and inspect the results. Make a guess of what will happen, before you actually specify the matrix in the applet.

\( a) \) \[
\begin{bmatrix}
0 & 1 \\
1 & 0 \\
\end{bmatrix}
\]
\( b) \) \[
\begin{bmatrix}
2 & 0 \\
0 & 1 \\
\end{bmatrix}
\]
\( c) \) \[
\begin{bmatrix}
0 & -1 \\
1 & 0 \\
\end{bmatrix}
\]
\( d) \) \[
\begin{bmatrix}
1 & -1 \\
1 & 1 \\
\end{bmatrix}
\]
\( e) \) your favorite matrix.

Exercise 4. (Bonus.) Recall the following exercise form the 5th Exercise Set.

The matrix \[
\begin{bmatrix}
7 & -1 \\
-2 & 8 \\
\end{bmatrix}
\]
describes a linear transformation \( f : \mathbb{R}^2 \to \mathbb{R}^2 \) in the standard basis of \( \mathbb{R}^2 \).

Let \( b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) and \( b_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \) be two vectors.

\( a) \) Compute the coordinate vectors of \( f(b_1) \) and \( f(b_2) \).

\( b) \) What is the matrix of \( f \) in the basis \( b = \{b_1, b_2\} \)?

Explore this problem via the applet from Exercise 3. (It is much more convenient to download the applet file from the website and opening it in GeoGebra (https://www.geogebra.org), than using the embedded applet in the browser.)

- Set \( A \) to be \[
\begin{bmatrix}
7 & -1 \\
-2 & 8 \\
\end{bmatrix}
\]

- On the left part of the screen, place the yellow, blue, and red corners of your square to \((1, 1)\), \((0, 3)\), and \((-1, 2)\), respectively. (Leave the green one in the origin.)

- What do you observe on the output field on the right-hand side?