Nonlinear Dynamics and Chaos

Homework 6

due, in class, Thursday January 11, 2018

1. Intermittency (2 pts.) Generate an orbit diagram of the logistic map

\[ x[n+1] = r x[n] (1-x[n]) \]  

for the parameter range \( r \in [3.825, 3.835] \). Identify the \( r \) value at which the dynamics change qualitatively, explain what happens at this parameter value by analyzing the dynamics of the appropriate iterate of the logistic map.

2. Kneading sequence of the Rössler return map

Construct the radial return map of the Rössler system as you did in homework 3 and make a spline interpolation to your data.

(a) (1 pts.) Find the critical point \( r_C \) of the radial return map.

(b) (3 pts.) Assign the binary symbols 1 and 0 respectively to the right and left hand sides of the critical point. Using this convention, find the first 10 digits of the kneading sequence and compute the kneading value.

(c) (2 pts.) Based on your finding, determine whether or not the following periodic orbits are admissible: 01, 100, 1011, 1001.

2. Hénon map

\[ (x_{n+1}, y_{n+1}) = f(x_n, y_n) = (y_n + 1.0 - ax_n^2, bx_n) \]  

is a map of the plane \((x, y)\) onto itself. Following Hénon, we are going to study this system at parameter values \( a = 1.4, b = -0.3 \).

(a) (1 pts.) Show that (2) is invertible and find \( f^{-1}(x_n, y_n) = (x_{n-1}, y_{n-1}) \).

(b) (1 pts.) Find the fixed points of (2) and compute their stability multipliers.

(c) (5 pts.) Let \( x^* \) be the fixed point of the Hénon system with stability multipliers \( \Lambda_1 > 1.0 > \Lambda_2 > 0 \) with corresponding stability vectors \( V_1 \) and \( V_2 \). A numerical approximation to the unstable manifold of \( x^* \) can be obtained by forward iterating the initial conditions

\[ x_0(\delta) = x^* + \epsilon \Lambda_1^\delta V_1, \]  

where \( \epsilon \) is a small constant and \( \delta \in [0,1) \) parametrizes the initial conditions. Similarly, the numerical approximation of the stable manifold can be computed as backwards iterations of the initial conditions

\[ x_0(\delta) = x^* + \epsilon \Lambda_2^{-\delta} V_2. \]
Set $\epsilon = 10^{-6}$, use 100 equidistant points for $\delta$, and iterate initial conditions (3) and (4) forwards and backwards respectively in order to approximate the unstable and stable manifolds of the fixed point. Plot these points for the final iteration times $N_f = 2, 10, 12, 15$ (forwards and backwards). You will find out that some of these trajectories will go to infinity, therefore plot only the points that are in the region $x \in (-1.5, 1.5)$, $y \in (-0.5, 0.5)$. Use different colors for the members of the stable and unstable sets. Based on your observations, answer the following question:

How many periodic orbits does the Hénon system (2) have?