Problem 1: Show that if \( f_\infty \) is the \( \Gamma \)-limit of some sequence of functions \( f_j \), then \( f_\infty \) is lower semicontinuous.

Problem 2: Construct a sequence of functions such that the pointwise and \( \Gamma \)-limits exist and are different at every point.

Problem 3: Show that if the \( \Gamma \)-limit \( f_\infty \) of a sequence of functions \( f_j \) exists, it is given by
\[
f_\infty(x) = \sup_{U \ni x} \liminf_{j \to \infty} \inf_{y \in U} f_j(y)
\]
where the supremum is over all neighborhoods \( U \) of \( x \).

Problem 4: As in the lecture, consider the functions
\[
f_j(u) = \int_0^1 a(jx)|u(x)|^2 dx
\]
where \( a \) is a smooth, positive and 1-periodic function. Consider the functions \( f_j \) to be defined on \( L^2([0, 1]) \) with the strong topology, i.e., the one generated by the \( L^2 \)-norm. Show that the \( \Gamma \)-limit of \( f_j \) coincides with the pointwise limit in this case.

Problem 5: Show the properties of lower-semicontinuous functions claimed in the lecture:

(i) \( f \) is lower semicontinuous iff \( \{ f \leq t \} \) is closed for all \( t \in \mathbb{R} \).

(ii) the supremum of a family of lower semicontinuous functions is lower semicontinuous

(iii) the characteristic function \( \chi_E \) is lower semicontinuous iff \( E \) is open.

Problem 6: Let \( f_j \) be lower semicontinuous functions such that \( f_j \leq f_{j+1} \) and \( f_j \to f_\infty \) pointwise. Show that \( f_\infty = \Gamma \text{-lim} f_j \).