Problem 1: Design a Catapult

In this assignment you will use your simulation code and some optimization approaches to construct a 2-dimensional catapult to throws its stone as far as possible. In one week we will have a session where everyone shows off their current best catapults and shortly describes the methods they used. In two weeks we will have our final competition to determine the winner. You may use the code provided by Christian Hafner to output actual simulation that will be used for comparison.

1.1 Problem setting

We start with the defining the catapult and its limitations, which is our design space.

1.1.1 Representation of a catapult

A catapult is described by the following parameters:

- Vertices represented by an nv-by-2 matrix containing vertex positions, where the $i$-th row contains the $x$ and $y$ coordinates of the $i$-th vertex;
- Edges represented by an ne-by-2 matrix containing vertex indices, where the $i$-th row contains the indices of the two vertices connected by the $i$-th edge;
- Spring thicknesses as a list of cross-sectional areas. The list has ne entries, one per edge. The thicker a spring, the stiffer it will be, but it will also use more material.

1.1.2 Limitations

Our catapult is constructed from elastic rods of different thicknesses and lengths. The rods are allowed to intersect each other, which mimics a 3D structure where the rods can be moving in different parallel planes. It is trivial to build an extremely large and powerful catapult, but in the real world you always face some limitations. In this subsection we list all of them:

- $x = 0$ is the border line not to be crossed by any node, meaning that for all nodes $x \leq 0$;
- $y = 0$ is the ground level and all nodes with $y = 0$ must be fixed, all nodes with $y > 0$ are free, and there are no nodes with $y < 0$;
- during the simulation no node or rod is allowed to go below $y = 0$;
- the dimensions of the catapult do not exceed $5 \times 5$ meters;
- total volume of all rods is 0.4 cubic meter;
- the minimal cross-section area for a rod is 0.001;
- you only have one material with the Young’s modulus $2 \cdot 10^6$ Pascal, which is about as stiff as very soft rubber;
- the density of the material is 1000 kg/m$^3$.
- the mass of the stone is 50 kg.
1.2 Catapult shooting

1.1.3 Other environment info

Additionally, the following applies:

- regular Earth gravity applies to the structure and the mass;
- the simulation system has some damping to prevent numerical instability.
- you can verify that the maximum weight of the catapult is 400 kg, derived from the known density and volume.

1.2 Catapult shooting

For a given valid catapult design we shoot it by first applying a constant force of the magnitude not exceeding 3000 N in any direction for not more than 10 seconds, then releasing it and only afterward releasing the stone at the time defined by you. Note that the resulting trajectory of the stone is fully determined by the velocity at the time when it was released, thus you can immediately estimate the length of its flight.

You need to provide the following data:

- the index of the vertex to which the force is applied;
- a force vector $f_{\text{load}}$ in Newton, $\|f_{\text{load}}\| < 3000$, with non-positive $x$ component $f_{\text{load},x} \leq 0$ (the force that pulls back the slingshot);
- a duration in seconds $t_{\text{load}} < 10$ for how long the force is being applied (winding up time);
- the index of the vertex to which the stone is attached;
- the duration in seconds $t_{\text{shoot}} > t_{\text{load}}$, after which the mass is released; at this point the position and velocity of the stone is determined and it will follow a parabolic trajectory.

See a schematic visualization of the shooting in Figure 1.

The goal is to throw the stone such that it lands as far as possible along the $x > 0$ axis.

1.3 Optimization of the catapult

To have a good starting point, you can draw a structure in the framework used for the previous homework, and save it. There is a utility function that allows you to load the vertex list and edge list from the saved file and use it as the simulation input. However, you still have to fill in the remaining inputs yourself. Look at the example to see which functions to call and how to visualize the result.

The simplest (in terms of coding) approach to optimize the design is to try out as many as you can manually and select the one which works best. Nevertheless, there are many more interesting approaches you could use to explore the design space. To beat your competitors, try to apply some of the methods presented at the lectures. Since the design space for our catapult is quite large and non-trivial, a good strategy would be to start optimizing only a few parameters for a given initial design and only when it works try to increase its complexity.
Figure 1: Schematic visualization of a valid catapult shooting.